Associate Professor WEN Zong-liang, PhD
E-mail: mathwzl@163.com
School of Management
Xuzhou Medical University
Associate Professor WU Xiaoli*, PhD, Corresponding Author
E-mail: wxiaoli@scut.edu.cn
School of Business Administration
South China University of Technology
Professor ZHOU Yong-wu, PhD
E-mail: zyw_666@hotmail.com
School of Business Administration
South China University of Technology

RETAILER'S OPTIMAL ORDERING POLICIES UNDER PARTIAL TRADE CREDIT

Abstract. *In this paper, we consider a dynamic inventory control problem where a retailer has limited cash flow and sells a single product to the market with random demand. In each period, the retailer can pay only for the partial amount of the purchased items when ordering and pays for the rest at the end of period. According to his initial inventory level and capital level, the retailer decides the fraction of immediate payment and order quantity so as to maximize his expected terminal cash at the end of the planning horizon. The retailer can either use his own capital or borrow a short-term loan from a lender to purchase the product, and the surplus cash (if any) can earn a risk-free interest. We employ the sequential optimization procedure to reduce the two-variable problem to an optimization problem over a single variable and present the structure of the retailer's optimal policy. Finally, numerical studies are given to demonstrate the model.*

Keywords*: inventory, dynamic, partial trade credit, financing.*

JEL Classification: C61

1. Introduction

Limited working capital is a frequent constraint in corporate procurement decisions. Trade credit is an important source of external financing, which is the most important form of short-term financing for firms in the United States, and is also used widely in both Europe and economies with less developed financial markets or weak bank-firm relationships (Booth et al., 2001; Wilson and Summers,

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2002). Trade credit (we call TC in the rest of the paper) is commonly used nowadays and brings benefits to the whole supply chain. First, TC can decrease the transaction cost or financial cost; help the upstream TC provider know better information of TC receiver. Second, as a type of price reduction, TC actually reduces the whole supply chain cost as well as default risk. Last but not the least, TC is certain kind of substitution to existing bank loan (Wang etc 2010). However, TC increases the default risk of the upstream supplier who provides TC.

In practice, in order to reduce the default risk, the supplier usually requires that the retailer pay for the partial amount of the purchased items immediately when ordering and for the rest of the purchased items can be settled at the end of trade credit period. This is so-called partial trade credit. Partial trade credit is widely used. For example, The Toyota Company offers partial delay payment to his downstream commission agent on the permissible credit period and the rest amount is paid at the time the replenishment order is placed. China's large appliances group Haier and Media provides partial TC to their distributors to reduce their capital shortage risk and encourage them to order more. Partial TC is commonly used in engineering projects and is compulsory in law. China's regulation of "Interim Measures for the Settlement of Construction Project Price" mentions: the advance payment for the contract work and materials project shall be paid according to the contract. In principle, the prepayment ratio shall not be less than 10% of the contract amount, not higher than 30% of the contract amount.

Nowadays finding reliable suppliers in the global supply chains has become so important for success (Rabbani et al., 2014) to achieve competition advantage and attract more orders as well as reduce default risks, and the suppliers frequently offer the retailer certain partial trade credit contract.

Given the partial trade credit ratio, Wen et al. (2014) consider a periodic-review inventory control problem where a capital-limited retailer sells a single product with random demand and is offered partial trade credit. They find that the retailer's optimal ordering strategy is either two-threshold policy or one-threshold policy depending on the retailer's initial working capital. However, if the supplier's power is not strong enough to force the retailer accept the payment fraction; instead, a larger retailer usually chooses the fraction to pay by now or later in business practice. For example, a powerful retailer, such as Wal-Mart, Apple and Vipshop (Chinese E-commerce company) etc., can determine the fraction of immediate payment. Thus, it is reasonable for the retailer to decide the fraction of immediate payment himself especially when the retailer is dominant retailer and the trade credit interest rate is large.

In this paper, we extend Wen et al.'s model (2014) to the case that the retailer can decide the fraction of immediate payment due to his current cash flow. We employ the sequential optimization procedure to reduce the two-variable problem to an optimization problem over a single variable and present the structure of the retailer's optimal policy. We find that the retailer will choose full trade credit under non-positive initial capital level; will choose partial trade credit for both

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relatively small initial capital level and not large inventory level; and will not choose trade credit for relatively large initial capital and mediate level of inventory; nevertheless, the retailer will not order at all for large initial inventory.

The remainder of this paper is organized as follows. We briefly review the related literature in Section 2. Section 3 focuses on the description and formulation of the basic model. Section 4.1 presents some preliminary results. In section 4.2 and section 4.3 we analyze the model and show the retailer's optimal policies under endogenous payment fraction. Section 5 conducts numerical experiments and section 6 concludes the paper with some suggestions on future research.

2. Literature review

Our work belongs to the interface of operations and finance, which has recently been paid fast-growing attention. We are mainly interested in the impact of partial trade credit on the inventory decision of a capital-limited retailer. There are two streams of related literature. One is related to trade credit or partial trade credit literature, the other is the multiple period inventory control literature relating operation and finance interface.

Most of the inventory management literature involved partial trade credit is based on the classical EOQ/EPQ framework, pioneering work by Goyal (1985). For instance, Taleizadeh (2013) studies an EOQ problem with partially delayed payment and partial backordering. Zhou et al. (2013) study the issue of how the retailer determines the optimal ordering policy and payment plan, where the retailer may choose to pay any fraction of the purchase cost within the short credit term and the rest must be paid within the long credit term. Some recent papers studied optimal order policies under the setting of two levels of trade credit. For example, Teng(2009) mentions that "in practice, a retailer frequently offers a partial down-stream trade credit to its credit risk customers". The retailer provides to his customer either partial trade credit or full trade credit. However, his model is based on EOQ model and does not consider stochastic demand, which is simpler than ours. Recently, Zhou et al. (2015) study a single-period inventory problem where the retailer faces stochastic demand and show the retailer's optimal strategy. They assume that the retailer can enjoy the partial trade credit from his supplier and borrow money from bank as well.

The growing literature on the interface between operations and finance mostly focus on single period problems. Some multi-period stochastic inventory models use simulation methods, e.g. Gocken (2017), Meng (2017). Only a few recent studies have considered dynamic inventory models with financial considerations. Two of these studies focus on the self-financing firms that solely rely on their internal capital to operate. Chao et al. (2008) study the optimal inventory policy of a self-financing firm to maximize its expected terminal wealth. The other studies incorporate one or more external financing sources into dynamic models. Li et al. (2013) consider a firm that makes production decisions, borrowing

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decisions and dividend policies for each period facing uncertain demand. The firm maximizes the expected present value of the infinite-horizon flow of the dividends. Xu and Birge (2006) propose a finite-horizon integrated planning model for a firm to maximize the expected discounted value of net cash flow to the firm's shareholders. With a similar objective to Xu and Birge (2006), under the assumption that firm continues to operate but pays a default penalty when bankrupt, Hu and Sobel (2007) study a multi-echelon inventory model and show that echelon base-stock policies are not optimal with financial constraints. In contrast to Chao et al. (2008), Gong et al. (2014) assume the firm can borrow short-term loans to finance its inventory in each period and study the firm's ordering policy to maximize its expected terminal cash.

3. Model description and formulation

We consider a dynamic inventory control problem with a risk neutral fund-limited retailer selling to the random market. The retailer makes replenishment decisions over a finite planning horizon of N periods, which are numbered 1 to N. The nonnegative demands D_n, $1 \leq n \leq N$, are independent and identically distributed (i. i. d.), with $f(\cdot)$ and $F(\cdot)$ being their probability density and cumulative distribution function, respectively. In the beginning of period n, the retailer purchases items from his supplier at unit price c, then sells the items at retail price p to customers. Any unfilled demand is lost. To avoid being trivial, we assume $p > c$ and lead time is zero. We assume zero salvage value.

We assume that the supplier has enough working capital, independent of the retailer's payment scheme. Hence, to achieve competition advantage and attract more orders as well as reduce default risks, the supplier offers the retailer partial trade credit contract, that is, the supplier charges the retailer a percentage, say β ($0 \leq \beta \leq 1$), of payment for items when the ordered items are delivered and allows the retailer to settle the unpaid account in the end of the period at an interest rate r_3, where β is decided by the retailer. We assume, the retailer can borrow freely from the bank at the interest rate r_2 (if needed). At the same time, the retailer can earn risk-free interest at the rate r_1 by investing surplus funds (if any). In this study, we assume $r_1 \leq r_3 \leq r_2$. The setting that trade credit interest is cheaper than bank loan is consistent with many empirical studies. For example, according to the sample of 1900 Italian manufacturing firms, Marrota (2005) reported that there is no evidence that trade credit is more expensive than bank loan. After surveying on 2500 Chinese firms, Fabbri and Klapper (2008) found that for over 20% of the firms surveyed, trade credit is cheaper than bank loans. Following the assumption in Hu and Sobel (2007) and Li et al. (2013), we assume the retailer can keep on operating with negative capital level. However, different from the two studies, we consider the case where the retailer does not have to pay a default penalty since negative one does not always lead to bankrupt in reality.

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The sequence of events is as follows. First, at the beginning of the planning horizon the supplier provides wholesale price c and partial trade credit interest rate r_3 to the retailer. At the beginning of each period, the retailer decides the order-up-to level and the immediate payment fraction, then pays for part of the payment, while pays the rest at the end of period under trade credit. There are two scenarios: 1) If the retailer's on-hand capital is enough to pay the part of instant payment, then he can invest the surplus capital to earn risk-free interest; 2) otherwise, he should seek funds from a bank to execute his procurement. Second, at the end of each period, demand is realized and the retailer receives his revenue from sales and salvage (if any), pays the unpaid amount as well as the trade credit interest to the supplier. Also the loan amount adding to interest owned to the lender will be returned if the second scenario happens. Moreover, we assume if the retailer's cash is not enough to repay the loan at the end, he can again borrow from the bank and leaves negative cash level on hand since he has collateral at bank.

Let b_n be the capital level, x_n and y_n be the inventory levels, before and after ordering, respectively. At the beginning of period $n < N$, the retailer borrows a loan$[b_n - \beta c(y_n - x_n)]^-$ from the bank at the rate r_2, pays the supplier $\beta c(y_n - x_n)$ up front, and invests the left funds $[b_n - \beta c(y_n - x_n)]^+$ at the risk-free rate r_1. At the end of the period, his wealth includes the sales revenue $p\min\{y_n, D_n\}$ and the return on investment $[b_n - \beta c(y_n - x_n)]^+ (1 + r_1)$. However, he has to pay off the rest purchase cost and partial trade credit interest $(1 - \beta)c(y_n - x_n)(1 + r_3)$ to the supplier and fulfill a financial obligation $[b_n - \beta c(y_n - x_n)]^- (1 + r_2)$ as well as the holding cost $h(y_n - D_n)^+$. The retailer's capital level at the end of period n, which is also the capital level at the beginning of period $n + 1$, is

$b_{n+1} = (p + h)\min\{y_n, D_n\} - hy_n - c(1 + r_3)(1 - \beta)(y_n - x_n) + \varphi(b_n - \beta c(y_n - x_n))$, (1)
where $n=1,2,\ldots,N-1$, $\varphi(s) = s(1 + r_2) + s^+(r_1 - r_2)$.

Since we assume lost-sale situation, the inventory level at the beginning of period $n+1$ is $x_{n+1} = (y_n - D_n)^+$ for $n=1,2,\ldots,N-1$. (2)

Therefore, given initial x_1 and b_1, the retailer will decide a replenishment strategy to maximize his expected terminal cash at the end of the planning horizon. Denote by $V_n(x, b)$ the maximum expected terminal cash given x and $b$at the beginning of period n. The optimality equation is

$$V_n(x,b) = \max_{\substack{y \geq x \\ 0 \leq \beta \leq 1}} E\left[V_{n+1}((y - D_n)^+, g(x,b,y,D_n)\right],$$

where$g(x, b, y,\beta, D_n) = (p + h)\min\{y, D_n\} - hy - c(1 + r_3)(1 - \beta)(y - x) + \varphi(b - \beta c(y - x))$,and$\varphi(s) = s(1 + r_2) + s^+(r_1 - r_2)$,with a condition $V_{N+1}(x, b) = b$.

For convenience, let $\pi_n(x, b, y, \beta) = E[V_{n+1}((y - D_n)^+, g(x, b, y, \beta, D_n))]$.

Next we will characterize the retailer's optimal policy.

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4. Model analysis

4.1. Preliminary results for exdogenous fraction of immediate payment

In this section, we briefly review the preliminary results and notations given in Wen et al.'s model (2014), which can be used here.

Lemma 1. (Monotonicity of value function)For any n and fixed x, $V_n(x, b)$ is increasing in b.

Lemma 2. For any n and given β, $V_n(A - z, B + (p + h)z)$ is increasing in z for fixed A and B.

Proposition 1. (Concavity of value function) For any n and given β, $V_n(x, b)$ is concave in (x, b).

When $b > 0$, for convenience, denote $R = x + b/(\beta c)$,whichrepresents the highest order-up-to level the retailer can afford by his own capital. We call R as the modified equity level. Let

$$\pi_n(x,b,y,\beta) \doteq \begin{cases} \pi_{1n}(x,b,y,\beta), & y \in [x,R], \\ \pi_{2n}(x,b,y,\beta), & y \in [R,+\infty), \end{cases}$$

and

$$\frac{\partial \pi_n(x,b,y,\beta)}{\partial y} \doteq H_n(y) \doteq \begin{cases} H_{1n}(y), & y \in [x,R], \\ H_{2n}(y), & y \in [R,+\infty). \end{cases}$$

Proposition 2.$\pi_n(x, b, y, \beta)$ is concave in (x, b, y) for given β, $\pi_{1n}(x, b, y, \beta)$ and $\pi_{2n}(x, b, y, \beta)$ are concave in y for given β and fixed (x, b).

Proposition 3. For $n = 1,2,\ldots,N$, there exist unique y_{1n} and y_{2n} so that$H_{1n}(y_{1n}) = 0$and$H_{2n}(y_{2n}) = 0$,respectively, and$y_{1n} \geq y_{2n}$.

Theorem 1. (Two-threshold type policy) For period n with given initial inventory level x and capital level b ($b > 0$) at the beginning of the period, there exist two thresholds y_{1n} and y_{2n}, which define the optimal order-up-to level $\bar{y}$ as follows:

$$\bar{y} = \begin{cases} \max\{x, y_{1n}\}, & \text{if } y_{1n} < R, \\ R, & \text{if } y_{2n} \le R \le y_{1n}, \\ y_{2n}, & \text{if } R > y_{2n}. \end{cases}$$

where y_{1n} and y_{2n} are identified by $H_{1n}(y_{1n}) = 0$ and $H_{2n}(y_{2n}) = 0$, respectively.

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Next, we present the optimal policiesunder the case $b \le 0$.

Theorem 2. (One-threshold type policy) For period n with given initial inventory level x and capital level b ($b \le 0$) at the beginning of the period, the optimal order-up-to level $\bar{y}$ is $\max\{x, y_{2n}\}$, where y_{2n} is uniquely identified by $H_{2n}(y_{2n}) = 0$.

Corollary 1. For any period n, the thresholds y_{1n} and y_{2n} are dependent of x and b. However, for period N,y_{1N} and y_{2N}, are independent of x and b.

Remarks.

Lemma 1 is intuitively clear: The more initial capital the firm has, the better it is to the firm’s terminal cash level. Lemma 2 is essential in proving the second-order property of the value function. The lemma says that it is better to keep cash than having inventory in stock. Theorem 1 and Theorem 2 characterize the retailer’s policy, and present the relationship between optimal order-up-to level $y(\beta)$ and fraction of immediate payment β.

4.2. Model analysis for endogenous fraction of immediate payment

In this model, β is not exogenously given by the supplier but determined by the retailer instead. Hence, the retailer determines both order quantity and immediate paymentfraction. We use the subscript “β” to distinguish the notations from previous model, if any. Denote the optimal inventory level and fraction of immediate payment by $\bar{y}_\beta$ and $\bar{\beta}$, respectively. Then, one has

$$(\bar{y}_\beta, \bar{\beta}) = \arg \max_{y \ge x, 0 \le \beta \le 1} \pi_n(x, b, y, \beta) . \tag{3}$$

In what follows, for simplicity, we suppress $\pi_n(y, \beta)$,$\pi_{1n}(y, \beta)$ and $\pi_{2n}(y, \beta)$ as the value of $\pi_n(x, b, y, \beta)$, $\pi_{1n}(x, b, y, \beta)$ and $\pi_{2n}(x, b, y, \beta)$ for fixed x and b, respectively. Recall that Theorem 1 and Theorem 2 characterize the relationship between $y(\beta)$ and β. Since $\pi_n(y, \beta)$is concave in y for given β, we can reduce the two-variable problem (3) to an optimization problem over single variable β by substituting $y(\beta)$ back into $\pi_n(y, \beta)$. The method is called sequential optimization procedure, which was used by Whitin (1955), Zabel (1970)and Petruzzi and Dada (1999).

It should be pointed out that when $b \le 0$, $\pi_n(y, \beta) = \pi_{2n}(y, \beta)$. Here, the relation between optimal order-up-to level and β is simple, that is, $y(\beta) = \max\{x, y_{2n}(\beta)\}$. By substituting $y(\beta)$ into $\pi_{2n}(y, \beta)$, we can transform the optimization problem (3) into a maximization problem over the single variable β, i.e., $\max_{0 \le \beta \le 1} \pi_n(y(\beta), \beta)$. However, when $b > 0$ the relationship between $y(\beta)$ and β is rather complex. As shown in Theorem 2, $y(\beta)$ is a piecewise function consisting of three sections. To employ sequential optimization procedure, we present the following

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preliminary results. The following Proposition 4 presents the monotonicity of $y_{1n}(\beta)$ and $y_{2n}(\beta)$.

Proposition 4. For any $0 \le \beta \le 1$, $y_{1n}(\beta)$ increases in β but $y_{2n}(\beta)$ decreases in β.

Proof. Take the first order derivative of $H_n(y(\beta))$ with respect to β, from the definition of $y_{1n}(\beta)$, $y_{1n}(\beta)$ and $H_n(y(\beta))$, we have

$$\frac{dH_n(y(\beta))}{d\beta} = c(y(\beta)-x)EV_{n+1}^2[(r_3-r_2)-(r_1-r_2)\mathbf{1}_{\{y<R\}}] = \begin{cases} c(y(\beta)-x)EV_{n+1}^2(r_3-r_2), & \text{if } y(\beta)<R, \\ c(y(\beta)-x)EV_{n+1}^2(r_3-r_1), & \text{if } y(\beta)\ge R. \end{cases}$$

Note $r_1 \le r_3 \le r_2$ and $V_{n+1}^2 > 0$ from Lemma 1, then H_{1n} decreases but H_{2n} increases in β, indicating y_{1n} increases and y_{2n} decreases in β.

Note that for any $0 < \beta \le 1$, $R(\beta) = x + b/(\beta c)$. If $b - \beta c(y_{in}(\beta) - x) = 0$, i.e., $y_{in}(\beta) = x + b/(\beta c)$, then $y_{in}(\beta) = R(\beta)$, which means $\pi_{in}(y_{in}(\beta), \beta) = \pi_{in}(R(\beta), \beta)$ for i= 1,2. Since $\pi_{1n}(R(\beta), \beta) = \pi_{2n}(R(\beta), \beta)$ due to the continuity of $\pi_n(y, \beta)$, for notational simplicity, in what follows we denote $\pi_{1n}(R(\beta), \beta)$ and $\pi_{2n}(R(\beta), \beta)$ as $\pi_n(R(\beta), \beta)$ when $y_{in}(\beta) = R(\beta)$. Next, we analyze the property of $\pi_n(R(\beta), \beta)$.

Taking the first-order derivative of $\pi_n(x, b, R(\beta), \beta)$ with respect to β,

$$\frac{d\pi_n(R(\beta),\beta)}{d\beta} = -\frac{b}{\beta^2 c}\left\{E\left[V_{n+1}^1\mathbf{1}_{\{R>D_n\}} + V_{n+1}^2\left[(p+h)\mathbf{1}_{\{R<D_n\}} - h - c(1+r_3)\right]\right]\right\}.$$

Where the superscript i denotes taking the first order partial derivative of V_{n+1} with respect to the ith variable for i=1,2.

Let $J_n(\beta) = \frac{d\pi_n(R(\beta),\beta)}{d\beta}$, we can easily obtain

$$\left.\frac{dJ(\beta)}{d\beta}\right|_{\frac{dR(\beta)}{d\beta}=0} = \left.\frac{d^2R(\beta)}{d\beta^2}\right|_{\frac{dR(\beta)}{d\beta}=0} < 0.$$

Hence, we have the following Proposition 5.

Proposition 5. $\pi_n(R(\beta), \beta)$ is unimodal for $0 < \beta \le 1$.

Proposition 5 plays an important role in the analysis of the retailer's optimal policy. To study the retailer's optimal policy, we show a preliminary lemma.

Lemma 3. For any $0 \le \beta \le 1$, the following results hold:
(1) If $x < y_{1n}(\beta)$, then $\pi_{1n}(y_{1n}(\beta), \beta)$ increases in β;
(2) If $x < y_{2n}(\beta)$, then $\pi_{2n}(y_{1n}(\beta), \beta)$ decreases in β.
Proof Substituting $y_{in}(\beta)$ into $\pi_n(y(\beta), \beta)$ and taking the first order derivative of $\pi_n(y_{in}(\beta), \beta)$ with respect to β, we obtain

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$$\frac{d\pi_n(y(\beta),\beta)}{d\beta}=c(y(\beta)-x)EV_{n+1}^2[(r_3-r_2)-(r_1-r_2)\mathbf{1}_{\{y<R\}}]=\begin{cases}c(y(\beta)-x)EV_{n+1}^2(r_3-r_2), & \text{if } y(\beta)<R,\\ c(y(\beta)-x)EV_{n+1}^2(r_3-r_1), & \text{if } y(\beta)\ge R.\end{cases}$$

Note that $r_1 \le r_3 \le r_2$ and $V_{n+1}^2 > 0$, from Lemma 1, if $x < y_{1n}(\beta)$, then $\frac{d\pi_{1n}(y_{1n}(\beta),\beta)}{d\beta}>0$, which means that $\pi_{1n}(y_{1n}(\beta), \beta)$ increases in β. Similarly, if $x < y_{2n}(\beta)$, $\pi_{2n}(y_{1n}(\beta),\beta)$decreases in β.

Lemma 3establishes some properties of $\pi_{in}(y_{in}(\beta), \beta)$. Following Proposition 5 and Lemma 3, we have Lemma 4.

Lemma 4. Let $L_{in}(\beta) = b - \beta c(y_{in}(\beta) - x)$ for $i = 1,2$. For any $0 \le \beta \le 1$, the following results hold:
(1) $L_{2n}(\beta) \ge L_{1n}(\beta)$, where “=” holds if and only if $\beta = 0$.
(2) If $x < y_{1n}(\beta)$, then $L_{1n}(\beta)$ decreases in β.
Proof (1) Lemma 4(1)follows from Proposition 3 and the definition of $L_{in}(\beta)$.
(2) Taking the first-order derivative of $L_{1n}(\beta)$, we obtain

$$\frac{dL_{1n}(\beta)}{d\beta}=-c\left[y_{1n}(\beta)-x+\beta\frac{dy_{1n}(\beta)}{d\beta}\right].$$

From Prop 4(2), we have $\frac{dy_{1n}(\beta)}{d\beta}>0$. Thus, $\frac{dL_{1n}(\beta)}{d\beta}<0$ for $x < y_{1n}(\beta)$.

From the definition of $y_{1n}(\beta)$ and$y_{2n}(\beta)$, we have $y_{1n}(0) = y_{2n}(0)$. To avoid confusing, in what follows at $\beta = 0$, we use $y_n(0)$ to represent $y_{in}(0)$.

Lemma 5.
(1) For any $0 < \beta \le 1$, if $x < y_n(0) - b/(\beta c)$, then $J_n(\beta) < 0$; otherwise, $J_n(\beta) \ge 0$.
(2) For any $0 < \beta_1 < \beta_2 < 1$, if $L_{1n}(\beta_1) = 0$ and $L_{2n}(\beta_2) = 0$, then $J_n(\beta_1) > 0$ and $J_n(\beta_2) < 0$, and β_0 is the maximum point of $\pi_n(x, b, R(\beta), \beta)$ for $0 < \beta \le 1$, where β_0 is the unique root in the interval (β_1, β_2) such that $J_n(\beta) = 0$.
Proof .(1) From the definition of $y_n(0)$, we have

$$E\left[V_{n+1}^1\mathbf{1}_{\{y_n(0)>D_n\}}+V_{n+1}^2\left[(p+h)\mathbf{1}_{\{y_n(0)<D_n\}}-h-c(1+r_3)\right]\right]=0.$$

If $x < y_n(0) - b/(\beta c)$, then $x + b/(\beta c) < y_n(0)$, i.e. $R < y_n(0)$, thus $J_n(\beta) < 0$ for any $0 < \beta \le 1$. If $x \ge y_n(0) - b/(\beta c)$, then $x + b/(\beta c) \ge y_n(0)$, i.e. $R \ge y_n(0)$, thus $J_n(\beta) \ge 0$ for any $0 < \beta \le 1$.

(2) If $L_{in}(\beta_i) = 0$, i.e., $b - \beta c\,(y_{in}(\beta) - x) = 0$, then$R = y_{in}(\beta)$. From the definition of $y_{in}(\beta)$ and $J_n(\beta)$, we have $J_n(\beta_1) > 0$ and $J_n(\beta_2) < 0$. It follows from Proposition 5 that β_0 is the maximum point of $\pi_n(x, b, R(\beta), \beta)$ for $0 < \beta \le 1$, where β_0 is the unique root in the interval (β_1, β_2) such that $J_n(\beta) = 0$.

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Lemma 6. For any closed set $I \subseteq [0, 1]$, let $\beta^l = \inf I$ and $\beta^u = \sup I$,
(1) if $y(\beta) = \max\{x, y_{1n}(\beta)\}$ and $x < y_{1n}(\beta^u)$, then $\pi_{1n}(y_{1n}(\beta), \beta) \leq \pi_{1n}(y_{1n}(\beta^u), \beta^u)$ for any $\beta \in I$;
(2) if $y(\beta) = \max\{x, y_{2n}(\beta)\}$ and $x < y_{2n}(\beta^l)$, then $\pi_{2n}(y_{2n}(\beta), \beta) \leq \pi_{2n}(y_{2n}(\beta^l), \beta^l)$ for any $\beta \in I$.

Proof. We first prove part (1) of Lemma 6, part (2) can be shown similarly.

When I is continuous, from part (2) of Proposition 4 we have $y_{1n}(\beta^l) \leq y_{1n}(\beta) \leq y_{1n}(\beta^u)$ holds for any $\beta \in I$. Since $x < y_{1n}(\beta^u)$, there are two possible cases: $x < y_{1n}(\beta^l)$ and $y_{1n}(\beta^l) \leq x \leq y_{1n}(\beta^u)$.

If $x < y_{1n}(\beta^l)$, then $x < y_{1n}(\beta)$ for any $\beta \in I$. Hence, we have $y(\beta) = \max\{x, y_{1n}(\beta)\} = y_{1n}(\beta)$ and $\pi_n(y(\beta), \beta) = \pi_{1n}(y_{1n}(\beta), \beta)$. From part (2) of Lemma 3, we know that $\pi_{1n}(y_{1n}(\beta), \beta)$ increases in β. Thus, $\pi_{1n}(y_{1n}(\beta), \beta) \leq \pi_{1n}(y_{1n}(\beta^u), \beta^u)$ for any $\beta \in I$, in other words, $\pi_n(y(\beta), \beta) \leq \pi_{1n}(y_{1n}(\beta^u), \beta^u)$.

If $y_{1n}(\beta^l) \leq x \leq y_{1n}(\beta^u)$, then there exists a unique $\beta_1^x \in [\beta^l, \beta^u]$ such that $y_{1n}(\beta_1^x) = x$ since $y_{1n}(\beta)$ increases in β, and β_1^x divides $[\beta^l, \beta^u]$ into two subsets $[\beta^l, \beta_1^x)$ and $[\beta_1^x, \beta^u]$. For $\beta \in [\beta^l, \beta_1^x)$, $x \geq y_{1n}(\beta)$, then $y(\beta) = \max\{x, y_{1n}(\beta)\} = x$, we have $\pi_n(y(\beta), \beta) = \pi_{1n}(x, \beta) = \pi_{1n}(x, \beta_1^x)$. For $\beta \in [\beta_1^x, \beta^u]$, $x < y_{1n}(\beta)$, then $y(\beta) = \max\{x, y_{1n}(\beta)\} = y_{1n}(\beta)$, and $\pi_n(y(\beta), \beta) = \pi_{1n}(y_{1n}(\beta), \beta)$. From part (2) of Lemma 3, $\pi_{1n}(y_{1n}(\beta), \beta)$ increases in β, then $\pi_{1n}(x, \beta_1^x) < \pi_{1n}(y_{1n}(\beta), \beta) < \pi_{1n}(y_{1n}(\beta^u), \beta^u)$ for any $\beta^l < \beta < \beta_1^x$. As a result, $\pi_n(y(\beta), \beta) \leq \pi_{1n}(y_{1n}(\beta^u), \beta^u)$ for any $\beta \in I$.

If I is not continuous, we can divide I into several continuous closed sets. Then, we can complete the proof of part (1) through similar method as we used above.□

4.3. Retailer's optimal policy

Following the preliminary lemmas, we can characterize the retailer's optimal policies in the following Theorem 3 and Theorem 4.

Theorem 3. For period n, given initial inventory level x and capital level b, when$b \leq 0$, if $x \geq y_n(0)$, the retailer's optimal policy is to order nothing; otherwise if $x < y_n(0)$, he will order up to $y_n(0)$ and choose full trade credit.

Proof. When $b \leq 0$, from Theorem 2 we have $y(\beta) = \max\{x, y_{2n}(\beta)\}$. From Proposition 4 (1), $y_n(0) \geq y_{2n}(\beta) \geq y_{2n}(1)$ holds for any β. Thus, if $x \geq y_n(0)$, then $x \geq y_{2n}(\beta)$. Hence we have $\bar{y}_\beta = x$. If $x < y_n(0)$, from Lemma 6(2), we have for anyβ, $\pi_{2n}(y_{2n}(\beta),\beta) \leq \pi_{2n}(y_n(0),0)$.Thus, we have $\bar{y}_\beta = y(0)$ and $\bar{\beta} = 0$. □

Theorem 4. For period n, given initial inventory level x and capital level b, when$b > 0$, the retailer's optimal policy depends on both x and b. That is,

1) if$x \geq y_{1n}(1)$, the retailer orders nothing and, hence, has no payment;

2) for $y_{1n}(1) - b/c \leq x < y_{1n}(1)$, the retailer's optimal ordering policy is to order up to a high threshold $y_{1n}(1)$ and invest the rest capital;

3) for $y_n(0) - b/c \leq x < y_{1n}(1) - b/c$, the retailer will use up all his asset and order up to $R(1)$;

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4) if $0 \le x < y_n(0) - b/c$, the optimal order-up-to level is $R(\beta_0)$ and the optimal fraction of immediate payment is β_0, where $0 < \beta_0 < 1$ is the unique root to equation $J_n(\beta) = 0$.

Proof. When $b > 0$, from Proposition 4, we have $y_{1n}(1) \ge y_{1n}(\beta) > y_n(0)$ and $y_{2n}(\beta) < y_n(0)$ for any $0 < \beta \le 1$.

1) If $x \ge y_{1n}(1)$, we have $x \ge y_{1n}(\beta)$. According to Theorem 1, we have $y(\beta) = x$, which gives $\bar{y}_\beta = x$, i.e. the retailer does not need to order and determine β.

2) and 3)If $y_n(0) - b/c \le x < y_{1n}(1)$, from Lemma 5 (1) we have $J_n(1) \ge 0$. Due to Proposition 5, $\pi_n(R(\beta), \beta)$ increases in β for $0 < \beta \le 1$. For $y_n(0) - b/c \le x < y_{1n}(1)$, we show through two cases: $x \ge y_{1n}(1) - b/c$ and $x < y_{1n}(1) - b/c$.

1) If $x \ge y_{1n}(1) - b/c$, then $L_{1n}(1) \ge 0$. Next, we consider two subcases: $x < y_n(0)$ and $x \ge y_n(0)$.

When $x < y_n(0)$, $x < y_{1n}(\beta)$. Thus, $L_{1n}(\beta)$ decreases in βfromLemma 4(2). Hence, $L_{1n}(\beta) \ge L_{1n}(1) \ge 0$ for any $0 \le \beta \le 1$. Then, from Theorem 1, we have $y(\beta) = \max\{x, y_{1n}(\beta)\} = y_{1n}(\beta)$. Since $\pi_{1n}(y_{1n}(\beta), \beta)$ increases in β when $x < y_1(\beta)$, it is natural to have $\bar{\beta} = 1$and $\bar{y}_\beta = y_{1n}(1)$.

When $x \ge y_n(0)$, $x \ge y_{2n}(\beta)$. Thus, $L_{2n}(\beta) \ge b > 0$ for $0 \le \beta \le 1$. By Theorem 1,

$$y(\beta) = \begin{cases} \max\{x, y_{1n}(\beta)\}, & \text{if } L_{1n}(\beta) > 0, \\ R(\beta), & \text{if } L_{1n}(\beta) \le 0. \end{cases}$$

In what follows, our analysis is based on whether or not $L_{1n}(\beta) > 0$ holds.

(i) If $L_{1n}(\beta) > 0$ holds for any $0 \le \beta \le 1$, then $y(\beta) = \max\{x, y_{1n}(\beta)\} = y_{1n}(\beta)$. Since $\pi_{1n}(y_{1n}(\beta), \beta)$ increases in β when $x < y_{1n}(\beta)$, we have $\bar{\beta}$ =1and $\bar{y}_\beta = y_{1n}(1)$.

(ii) If there exists a $\beta \in (0, 1]$ such that $L_{1n}(\beta) \le 0$, then there exist at least two roots in the interval (0,1] such that $L_{1n}(\beta) = 0$ since $L_{1n}(0) > 0$ and $L_{1n}(1) > 0$. Let β_1 be the biggest root, then $L_{1n}(\beta) \ge 0$ for any $\beta_1 \le \beta \le 1$. On one hand, from Lemma 6(1), we have $\pi_n(y(\beta), \beta) \le \pi_{1n}(y_{1n}(1),1)$ for any $\beta \in \{\beta \mid L_{1n}(\beta) \ge 0, 0 \le \beta \le 1\}$, which means $\pi_n(y(\beta_1),\beta_1) \le \pi_{1n}(y_{1n}(1),1)$. On the other hand, since $\pi_n(R(\beta), \beta)$ increases in β for $0 < \beta \le 1$, we have $\pi_n(R(\beta), \beta) \le \pi_n(R(\beta_1),\beta_1)$ for any $\beta \in \{\beta \mid L_{1n}(\beta) \le 0, 0 < \beta \le \beta_1\}$. Recall that $\pi_{1n}(y_{1n}(\beta_1), \beta_1) = \pi_n(R(\beta_1), \beta_1)$, we have $\pi_n(y(\beta), \beta) \le \pi_{1n}(y_{1n}(1),1)$ for any $0 < \beta \le 1$. It follows that $\bar{\beta}$ =1and $\bar{y}_\beta = y_{1n}(1)$.

To sum up, if $y_1(1) - b/c \le x < y_1(1)$, then $\bar{y}_\beta = y_{1n}(1)$ and $\bar{\beta}$ =1.

2) If $x < y_{1n}(1) - b/c$, then $L_{1n}(1) < 0$. It follows that there exists at least one root in the interval (0, 1) such that $L_1(\beta) = 0$. Let β_1 be the biggest root, then $L_{1n}(\beta) < 0$ for any $\beta_1 < \beta \le 1$. On one hand, by Lemma 6(1), we have $\pi_n(y(\beta), \beta) \le \pi_{1n}(y_{1n}(\beta_1), \beta_1)$ for any $\beta \in \{\beta \mid L_{1n}(\beta) \ge 0, 0 \le \beta \le \beta_1\}$. On the other hand, since $\pi_n(R(\beta), \beta)$ increases in β for $0 < \beta \le 1$, we have $\pi_n(R(\beta), \beta) \le \pi_n(R(1),1)$ for any $\beta \in \{\beta \mid L_{1n}(\beta) \le 0, 0 < \beta \le 1\}$.

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Recall that $\pi_{1n}(y_{1n}(\beta_1), \beta_1) = \pi_n(R(\beta_1), \beta_1)$, we have $\pi_n(y(\beta), \beta) \leq \pi_n(R(1),1)$ for any $0 \leq \beta \leq 1$. It follows that $\overline{\beta} =1$ and $\overline{y}_\beta = R(1)$.

Therefore, if $y_n(0) - b/c \leq x < y_{1n}(1) - b/c$, then $\overline{y}_\beta = R(1)$ and $\overline{\beta} =1$.

If $0 \leq x < y_n(0) - b/c$, then from Lemma 5 (1) we have $J_n(1) <0$. Since $x < y_n(0) - b/c$ and $y_n(0) < y_{1n}(1)$, we have $x < y_{1n}(1) - b/c$, i.e., $L_{1n}(1) <0$. As mentioned above, $x < y_n(0)$ implies that $L_{1n}(\beta)$ decreases in β for any $0 \leq \beta \leq 1$. Hence, there exists a unique $\beta_1 \in (0, 1)$ such that $L_{1n}(\beta) = 0$. Thus, $L_{1n}(\beta) \geq 0$ for any $0 \leq \beta \leq \beta_1$. Then, by Lemma 2, we have $y(\beta) = \max\{x, y_{1n}(\beta)\}$. It follows that $\pi_n(y(\beta), \beta) \leq \pi_{1n}(y_{1n}(\beta_1), \beta_1)$ for any $0 \leq \beta \leq \beta_1$ (due to part (1) of Lemma 6).

The case of $\beta_1 \leq \beta \leq 1$ can be similarly shown and we omit here.

Hence, if there exists a $\beta \in [\beta_1,1]$ such that $L_{2n}(\beta) \leq 0$, then $\overline{y}_\beta = R(\beta_0)$ and $\overline{\beta} = \beta_0$.□

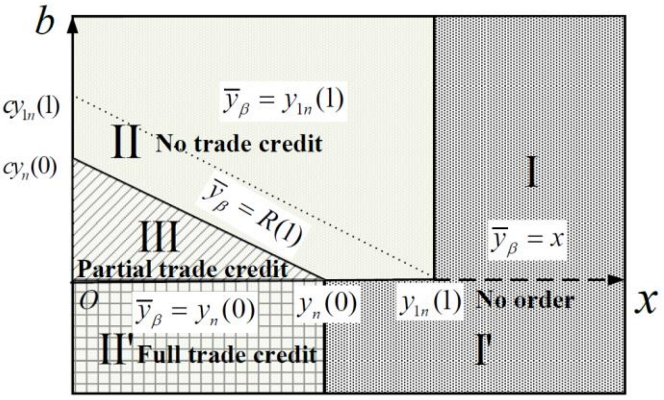

Figure 1.Retailer's optimal policy for endogenous immediate payment

Theorem 4 describes the optimal ordering policies for b>0. $x \geq y_{1n}(1)$ represents area I in Fig 1, meaning no ordering for larger initial inventory level. For any initial inventory level x satisfies $y_n(0) - b/c \leq x < y_{1n}(1)$ (area II in Fig 1), immediate payment in full is the best option for the retailer. And for $0 \leq x < y_n(0) - b/c$(area III in Fig 1), partial payment is the best option for the retailer. In this case, the optimal policy is to use up his working capital without borrowing.

As shown in Figure 1, for positive initial capital level, there exist three regions I, II and III, which represent the retailer's *no order*, *no trade credit* and *partial trade credit* regions, respectively. For negative initial capital level, there are two regions I' and II', denoting the retailer's *no order* and *full trade credit* regions, respectively. If initial state (x,b) falls into*no order* region, the retailer's inventory level is relatively high (greater than the higher threshold $y_{1n}(1)$ when $b > 0$ or greater than the lower threshold $y_n(0)$ when $b \leq 0$). Hence, it is optimal for the retailer not to order. If (x,b) belongs to *no trade credit* region, the retailer will pay in full immediately when ordering. When the retailer's initial equity level $R(1)$ is relatively high (say, greater than $y_{1n}(1)$), he orders up to$y_{1n}(1)$ and invests the

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surplus capital to earn risk-free income. As his equity level is between the two thresholds $y_n(0)$ and$y_{1n}(1)$, he will use up all his on-hand capital to order up to $x + b/c$. If the initial state (x,b) is within *partial trade credit* region, the retailer's initial equity level is relatively small (i.e., lower than $y_n(0)$), he will order up to $y_n(0)$ and pay the fraction β_0 of the ordered items by using up capital and borrowing from a third party financial institution.

An interesting managerial insight from the analysis is that facing with imperfect information about retailer's initial b, suppliers can design the trade credit interest rate (i.e. $r_1 \le r_3 \le r_2$) to elicit information on creditworthiness. For example, once the penalty level is relatively moderate, the retailer will choose full trade credit if orderunder non-positive initial capital;and will choose no trade credit or partial trade credit and pay to the supplier as much as possible when order for positiveinitial capital level.In this regard, the retailer's payment decision reveals his cash status to some extent.

5. Sensitivity analysis on fraction of immediate payment for the last period

Due to the complexity of the problem considered, we numerically illustrate the impacts of the retailer's initial capital level and inventory level on the optimal fraction of immediate payment for the last period. Suppose that the demand has truncated normal distribution♦ with variance 9 and mean 8 and 16, respectively. The other parameters are $c = 1$, $p = 2.5$, $h = 0$, $r_1 = 0.05$, $r_2 = 0.2$, $r_3 = 0.15$.

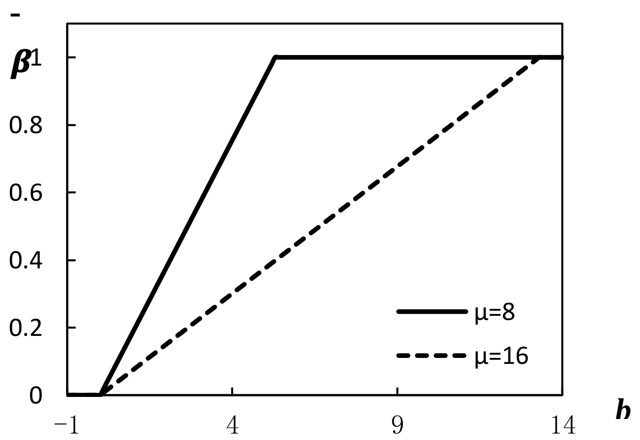

Figure 2.Impact of retailer's capital level on $\bar{\beta}$.

♦ We also use exponential distribution and uniform distribution to perform these experiments. The similar conclusions can be obtained. For brevity, we only take the truncated normal distribution as an example to show the results.

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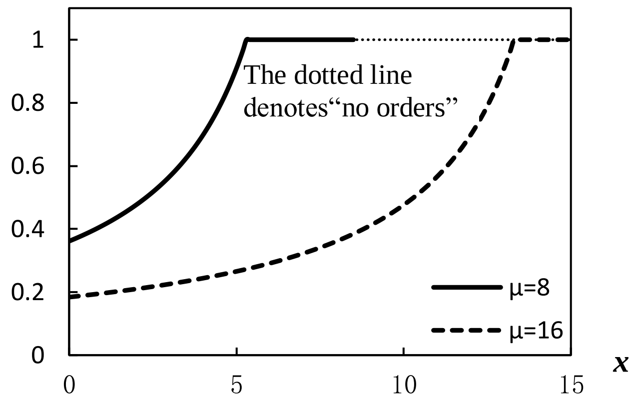

Figure 3.Impact of retailer's inventory level on $\bar{\beta}$.

First, setting the retailer's initial inventory $x = 0$,we examine how initial capital level affects the optimal fraction of immediate payment $\bar{\beta}$. The results are given in Fig.2. Fig.2 indicates that when the initial capital level b is negative, $\bar{\beta}$ is always equal to 0. That is, the retailer always chooses to pay for all the purchased items in delay. It is natural since the retailer has no cash on hand to support immediate payment. When the initial capital level is positive, $\bar{\beta}$ increases linearly from 0 up to 1 with the continuous increase of b. This further verifies that the retailer always prefers to finance his inventory with his own cash.

Setting the initial capital $b = 3$, we will examine how x affects the optimal $\bar{\beta}$. The results are given in Fig 3. From Fig. 3, we can see that the optimal $\bar{\beta}$ increases with x. It can be understood easily. Because the order-up-to level keeps unchangeable when x varies, the increase of x will leads to the decrease of practical order quantity. Thus, the same initial capital can pay for a higher fraction of the ordered items. Compared to initial capital, $\bar{\beta}$ is more sensitive to initial inventory.

The comparison of the mean at 8 and 16 in Fig.2 (Fig.3) also suggests that the optimal fraction of immediate payment is more sensitive to initial capital (inventory) for small size market than big size market.

6. Conclusion

As different trading partners have different financial situation, such as the level of cooperation and negotiation power etc., partial trade credit is widely used in practice. In this paper, we generalize Wen et al.'s model to the situation that the fraction of instant payment is endogenously determined by the retailer. We employ the sequential optimization procedure to reduce the two-variable problem to a single variable model and present the retailer's optimal policy structure.

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It should be mentioned that the computation involved thresholds policy is rather complex, and it is desirable to find an efficient heuristic algorithm. Also, terminal penalty may influence the effectiveness of partial trade credit in different directions. For example, the terminal penalty may weaken the risk-sharing role of partial trade credit. We can study the influence of terminal penalty on the optimal policy in future research. Other directions can be explored. For example, it might be worthy to consider a more complex but realistic trade credit interest form, such as asset-based rate, which is typical in many applications. In addition, we assume risk neutral decision making here, analyzing the retailer's risk performance is an interesting future research direction.

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